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SOME ARGUMENTS FOR THE OPERATIONAL READING OF TRUTH EXPRESSIONS

Abstract

The main question of our article is: What is the logical form of statements containing expressions such as “... is true” and “it is true that ...”? We claim that these expressions are generally not used in order to assign a certain property to sentences. We indicate that a predicative interpretation of these expressions was rejected by Frege and adherents to the prosentential conception of truth. We treat these expressions as operators. The main advantage of our operational reading is the fact that it adequately represents how the words “true” and “truth” function in everyday speech. Our approach confirms the intuition that so-called T-equivalences are not contingent truths, and explains why they seem to be—in some sense—necessary sentences. Moreover, our operational reading of truth

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expressions dissolves problems arising from the belief that there is some specific property—truth. The fact that we reject that truth is a certain property does not mean that we deny that the concept of truth plays a very important role in our language, and hence in our life. We indicate that the concept of truth is inseparable from the concept of sentence and vice versa—it is impossible to explicate one of these concepts without appeal to the other.

Keywords: truth, operator, predicate, logical form, Frege, Wittgenstein, Tarski

I

The question of truth causes many conflicts, even wars. Philosophy, in general, is not an exception to this rule. The difference is that philosophical struggles are wars of words. The majority of philosophers, however, agree on one question regarding the notion of truth. According to them, truth is a property and the expressions "... is true" and "it is true that ..." are predicates. This point of view is shared by the proponents of many conceptions of truth. Some identify the property of truth with correspondence to reality; others see it as long-run utility in a broad sense or consistency of an extensive set of beliefs.

What leads to the acceptance of the view that truth is a property and the expressions "... is true" and "it is true that ..." are predicates? It seems that the main reason is a belief that when we attach the word 'true' (or its derivatives) to some sentence¹ (or its name or description), we attribute to the latter some more or less abstract feature, namely being true. This feature amounts to, e.g., the sentence's relation to a mind-independent reality, or its coherence with other sentences accepted as true.

This view seems to be confirmed by grammar. The phrase "... is true" in sentences like "the sentence 'grass is green' is true" or "Fermat's last theorem

¹ We are aware of the tradition of recognizing the difference in meaning between the terms 'proposition' and 'sentence.' This difference is recognised in many ways in the literature. However, as we reject the existence of ideal entities called 'propositions,' so we do not use this word in the technical sense. Generally, we use the word 'sentence' to mark meaningful expressions; however, we do not exclude the possibility of the existence of sentences which lack meaning. In the latter case, we make it explicit. (It is worth noticing that the proposition/sentence difference does not appear in every European language, particularly in German.)

is true” functions as a predicative expression. The same can be said of the phrase “it is true that ...” in sentences like “it is true that snow is white” or “it is true that there are no elves.” From a grammatical point of view, truth expressions are predicates attributed to a subject anaphorically referred to by the pronoun ‘it.’ Hence, if we agree that standard grammatical analysis of these sentences gives us an accurate account of their logical form—at least in the respect being considered – then we must recognise the phrases “it is true that ...” and “... is true” as predicates.

Obviously, many of the proponents of such an account of truth are aware of the exceptional nature of the “is true” predicate and the uniqueness of the property of being true. This is evident by the fact that, according to them, truth can be characterised by sentences which are substitutions of Tarski’s Convention T². The equivalence “the sentence ‘April is the cruellest month’ is true iff April is the cruellest month” can serve as an example of such a substitution. According to the semantic theory of truth, the expression which attributes the property of truth to the sentence “April is the cruellest month” is logically equivalent to this sentence. So “is true” is quite an unusual predicate: It does not determine either any syntactic feature of sentences or, seemingly, their sense.³ In short, any sentence is what it is regardless of being true or false. However, on the other hand, when we find out which sentences are true and which are false, we gain real knowledge, the knowledge about the world. So, there is a vital difference between true and false sentences and it seems that the most natural way to express it is to say that truth and falsity are mutually exclusive properties of sentences.

Such intuitions as presented above lie at the core of the view that the expressions “it is true that ...” and “... is true” are predicates, despite their being fundamentally different from other predicates. Certainly, some of these

² Such characterisation of this predicate requires either a list of all T-equivalences or a definition which results in all T-equivalences.

³ Intuitively, the sense of any sentence must be independent of its being true: We need to know what it means to be able to recognise whether it is true or false. Cf. Wittgenstein (1961: 4.061–4.064). This view can be rejected assuming that a class of models for a given language can be determined by ascribing logical values to sentences of the given language (still not interpreted). However, this solution is exposed to the following objection: Truth predicates (“... is true”, “... is false”) in their normal use can be attributed to meaningful expressions only.

intuitions are right, but we believe that they do not entail the predicative character of truth expressions.

Strictly speaking, we do not propose a total rejection of the predicative theories of truth,⁴ because we can see neither a priori nor a posteriori reasons for the belief that the expressions "... is true" and "it is true that ..." cannot serve as predicates on some occasions due to the will of language users. For certain needs, it can be useful to accept that, e.g., for a given formal symbolic system L with a given syntax, it is possible to define certain feature of syntactically acceptable formulas, which can be named, for instance, 'model-related truth' (or 'model-related falsity,' alternatively). The definition of the feature and the predicate related to it would belong to a wider symbolic system, ML .

Now we must make one remark concerning the concepts of formal semantics and meta-language. Philosophers who accept a model theory think that the semantics of a given language L can be described in its meta-language, ML . ML enables us to define a model which is a structure consisting of a certain domain and a specific function by means of which a logical value can be assigned to every sentence of L . The meta-language may serve to represent the object language's sentences and the reality described by those sentences. It is possible because the so-called semantic definition of truth contains a set of translation rules which enable the rendering of each primitive expression of L into ML . So, one can give a precise definition of truth for L in ML . However, such a definition of truth for L is—from the perspective of ML —in a sense, purely syntactic. All it really says in regard to any given sentence p belonging to L is that it has the property of being true iff a certain sentence π belonging to ML which is a translation of p is true. That π is true is, however, taken for granted in ML , and that is the reason to claim that Tarski's definition is in some sense syntactic – the specification of true sentences of L is made by means of giving a list of true sentences of ML , and the sentences on the list play the role of axioms. The syntactic character of formal semantics has been noticed by the noblest critic of the whole idea of meta-languages, Ludwig Wittgenstein.⁵ Wittgenstein, in his later philosophy, emphasised that semantics cannot be derived from syntax.

⁴ We apply the term "predicative theory" to any conception which treats expressions like "it is true that ..." and "the sentence '...' is true" as predicates.

⁵ Cf. Wittgenstein (1979a: 133).

Understanding language requires not only a competence in construction and transformation of expressions, but also a competence in using them in different language-games. “Every sign *by itself* seems dead. *What* gives it life? —In use it is *alive*.”⁶

II

There is a fundamental difficulty with the account of truth as the property referred to by the predicate “... is true.” It concerns how to understand T-equivalence. If the expression “... is true” is a predicate ascribed to sentences, then T-equivalences seem to be non-trivial, which is counter-intuitive. Consider, for instance, the following equivalence:

1. The sentence “grass is green” is true iff grass is green.

That a certain object, namely the sentence “grass is green,” has a certain property (specifically, being true) neither follows from the fact that another object (grass) has another property (namely, being green), nor entails this fact. Some of the adherents of Tarski’s conception agree with that and accept the claim that T-equivalences are material. It seems, however, that Convention T imposes some additional constraints which prevent us from counting certain sentences as T-equivalences. Consider the following example:

2. The sentence “grass is green” is true iff Tarski emigrated to the USA.

This sounds absurd; therefore, we are forced to accept further constraints which will exclude merely material equivalences. The sentence on the right side of the equivalence must be a meta-linguistic translation of the sentence whose name occurs on the left side. Hence, according to the semantic definition of truth, the equivalence in 1 is non-trivial in itself, but only if such additional constraints are satisfied.

However, the language users intuitively recognise 1 and other sentences of its kind as necessarily and unconditionally true. Moreover, it seems that they do it because of the form of such sentences, not because of some feature of their content. This simple fact has been omitted due to the misconception of the function of quotation marks, as has been pointed out by Peter Geach.⁷ That some expression is mentioned does not exclude its being used at the

⁶ Wittgenstein (1974: §432).

⁷ Cf. Geach (1971: 81–85, 96).

same time. In contrast, we believe that mentioning an expression generally entails that it is being used.

This account of quotation marks can be proved by appeal to ordinary language intuitions. Note that we normally see no substantial difference (unless we consider superficial linguistic forms) between 1 and the following:

3. It is true that grass is green iff grass is green.⁸

According to Alfred Tarski, the logical form of 3 as an example of T-equivalence looks like this: “ x is a true sentence iff p .”⁹ Such an account of the form, if it is not supplemented in some way, does not allow us to infer the right side from the left side and *vice versa*. The inference—as Tarski says—can be made provided that the definition of truth is given.

III

We believe that Tarski’s interpretation of the logical form of 3 (and other utterances containing the phrases “... is true” or “it is true that ...”) presented above is wrong.¹⁰ There is another, much more accurate account of truth expressions. According to us, the logical role of phrases like “... is true” and “it is true that ...” is operational – they should be understood as operators.

The notion of an operator can be explained in terms of the notion of operation. If a symbol of a base of an operation is removed from a complex symbol of a result of this operation, what is left is a symbol of an operator. (For instance, if the base of our operation consists of the sentential symbols ‘ p ’ and ‘ q ,’ and the symbol of its result is ‘ $p \& q$,’ then the symbol ‘ $\&$ ’ is our operator.) We can explain the meaning of the two unary truth-operators using the following truth tables:¹¹

⁸ Elizabeth Anscombe makes a similar remark. Cf. Anscombe (2000: 4).

⁹ Cf. Tarski (1983).

¹⁰ This belief has been uttered previously by Peter Geach: “The genuine significance of ‘true’ is thus missed, and then queer theories and definitions of truth are framed.” Geach (1971: 96).

¹¹ Truth tables as complex symbols require further analysis. What is meant by ‘T’ and ‘F’ used in the tables should be explained in particular. For the time being, we naively treat the tables as “semantic” definitions of logical connectives. We shall reject this assumption soon.

p	$\sim p$
T	F
F	T

p	$\vdash p$
T	T
F	F

The first of them is certainly the well-known operator of logical negation which expresses the logical aspect of utterances like “it is not true that ...” and “... is false.” The other is the assertion operator introduced to modern logic by Frege in his famous *Begriffsschrift*.¹² According to the operational account, the logical form of 3 should be represented by the following formula: “ $\vdash p \equiv p$.”¹³ It should be noted that there is no need for any additional ‘definition of truth’ here, since we can infer the right side from the left side and *vice versa* by appealing only to the meaning of the assertion operator.

We cannot support our denial of predicative character of truth expressions by the authority of any classical philosopher. There are some traces of similar intuitions in the works of Frege and Wittgenstein, but even these two thinkers never clearly questioned the predicative account of truth. In more recent times, Dorothy Grover (and other prosententialists) and Robert Brandom explicitly rejected the latter view.¹⁴ However, we can point out

¹² Cf. Frege (1970: 11–12). It is worth mentioning that, according to Michael Dummett, the sign of assertion does not play the same role as the expression “it is true that ...” One of his arguments is that when we utter conditionals like “if it is true that it is raining, then streets are wet,” we assert neither that it is raining nor that streets are wet. According to Dummett, only a content stroke can be recognised as a sentential operator (though not without qualifications), not a sign of assertion. Cf. Dummett (1973: 295–363). The opposite suggestion has been made by van Heijenoort. Cf. Heijenoort (1967: 440–446). However, the accuracy of our account of the logical form of the utterances “it is true that ...” and “... is true” does not depend on who is right in this dispute. Our acceptance of Dummett’s interpretation would require a change in terminology only.

¹³ Dummett takes a different point of view: The sentence “it is true that grass is green” should be read as a statement which attributes truth to a certain thought. We refer to this thought by the utterance “that grass is green.” So, according to Dummett, this sentence should be analyzed in the following way: “it is true (that grass is green)”; the phrase “it is true” is a predicate and the phrase “that grass is green” is an individual constant. Cf. Dummett (2000: 10–11).

¹⁴ Grover is a co-author of the so-called prosentential theory of truth, according to which the phrase “... is true” is a semantically dependent part of the expression “it is true.” The latter is called a *prosentence* due to the analogy to pronouns and proverbs, and it can occur after normal sentences. Cf. Grover, Camp, Belnap (1975). Brandom presents

one otherwise famous thread in Kant's thought which may be viewed as an astonishingly close parallel to our understanding of how the notion of truth works. Kant, arguing against the ontological proof, describes the function of the notion of being as follows:

Being is obviously not a real predicate, i.e., a concept of something that could add to the concept of a thing. It is merely the positing (*Position*) of a thing or of certain determinations in themselves. In the logical use it is merely the copula of a judgement.¹⁵

We believe that this description is generally correct: 'being' in its usual meaning cannot be taken as a real predicate. The same is correct in regard to 'truth' in its original and natural sense: it cannot be taken as a real predicate either. As attribution of existence to something says nothing about its character, so does the ascription of truth to a sentence say nothing about its syntax or content.

As we noted before, the concept of truth can be given a predicative sense. The same thing can be done with the concept of being, since it is possible to divide names into two classes according to whether there are bearers of these names or not. We can, for instance, include names of existing animals (e.g., horses, cows, hens, boars, lions and elephants) to the first class and include names of nonexistent animals (e.g., dragons, hippogryphs, unicorns, dinosaurs and dodos) to the second class. Having done such a classification, we may be driven to the thought that being is a kind of feature of certain creatures. Nothing makes this thought wrong; however, its context – this classification of names—is quite unusual. What is particularly unusual here is the meaning of the word 'feature' in the sentence, "being is a kind of feature of certain creatures." This use of the word 'being,' although odd, can be easily understood, but does not serve as a home for our old, familiar concept of being.

One must remember, of course, that there is a great dissimilarity between the two pairs of elements of language and reality we try to juxtapose, for a sentence is not a name and a fact is not an object. Therefore, being

a modification of the prosententialists' account. According to him, expressions like "... is true" serve as sentential operators. Cf. Brandom (1994: 299–305). The conception we defend in the present paper is very similar to Brandom's account.

¹⁵ Kant (1998: 567).

should not be linked to facts represented by sentences, and truth should not be linked to what is referred to by a name or a definition. We recognise the importance of this categorial difference. However, many see even greater dissimilarity here: they say that truth is predicated to the linguistic entities (sentences) and being is attributed to the elements of extra-linguistic reality (things). Hence the two concepts are placed on different levels of language: the latter is recognised as belonging to the so-called object-language, and the former to its meta-language. We try to undermine this claim at the end of our paper.

IV

Our belief that expressions like “it is true that ...” and “... is true” play, in principle, the role of an operator is, in the first place, supported by observations of their function (grammar)¹⁶ in ordinary language. To begin with the typical contexts of everyday life, in the cases when we replace ellipses by whole sentences, the latter form (e.g., “the sentence ‘he has lost his mind’ is true”) is much less frequent than the former one (e.g., “it is true that he has lost his mind”). Therefore, we assume that the phrase “it is true that ...” is basic and the phrase “... is true” can be treated as a rarely used equivalent reserved for some special purposes (e.g., for meta-language). The fact that use of the truth expression “it is true that ...” does not require putting a relevant sentence in quotation marks already suggests an operational interpretation of this expression:

4. (It is true that) p

But still, some may wonder whether the very existence of such expressions as “the sentence ‘grass is green’ is true” does not undermine the operational conception of truth. As we mentioned above, there is only a difference in style between the example just given and the utterance “it is true that grass is green.” Hence, according to our account, the phrase “the sentence ‘...’ is true” is an operator, too. The possibility of assigning truth to a foreign

¹⁶ We use this notion after the later Wittgenstein; therefore, it should not be confused with the traditional concept of grammar. As we noticed before, according to traditional grammatical analysis, truth expressions work as predicates. Our turn towards non-philosophical contexts does not mean that we take at face value a common sense view of how language is employed.

language's sentences does not undermine this solution. We have to assume only that the operator "the sentence '...' is true" plays, in some contexts, an additional function: the result of its applying to foreign language sentences is the translation of these sentences into English; for instance, the effect of applying the operator "the sentence '...' is true" to the German sentence "Der Schnee ist weiss" is logically equivalent to the following English expression: "it is true that snow is white".

Another issue we must examine is the question: How should we analyze statements such as "All he said is true"? No sentences are used or mentioned in constructions of this type. We propose to render the structure of the statement by the following formula:

5. For every p (if X said p , then (it is true that) p).

We think that in case of such judgments, quantification is substitutional and, therefore, their acceptance does not commit us to the belief that there are some entities referred to by the sentences; for example, *propositions* conceived as some abstract and extralinguistic things. By the way, the conception of ontological commitment is exposed to serious objections.¹⁷

Note that the phrases "it is true that ..." and "... is true"—as we consider their syntactic form—belong to the family of expressions: "it is possible that ...," "it is necessary that ..." and, obviously, "it is not true that ..." All these expressions are treated in logical calculus as operators. So, why should an exception be made for "it is true that ..."? Of course, it should not.

The operational interpretation is, to some extent, consistent with the redundancy theory, but on the other hand, the operational reading of phrases like "it is not true that ..." does not suggest they are redundant. On the contrary, their contribution to the meaning of a whole utterance is plainly obvious. Why? Because each time our use of negation is not pragmatically idle. We seem to move the action forward: We come to a different 'truth-possibility'¹⁸ which can be particularly spectacular in a context of other logical operators, especially quantifiers.

¹⁷ First, the famous *dictum*, "to be is to be a value of a variable," cannot be formulated in the canonical notation. Second, if we accept Frege's assumption that being is not a property of individuals, then Quine is wrong: We can say that Julius Caesar is a value of a variable, but we cannot say that he exists. Cf. Geach (1968: 161–162).

¹⁸ This concept was used by the early Wittgenstein. Cf. Wittgenstein (1961: 4.3).

Assertion behaves differently. Logicians quickly lost their interest in the assertion operator due to its logical ‘redundancy.’¹⁹ It is evident that this operator does not change the logical value of its argument and its effectiveness is equal to the practical effectiveness of exponentiation with exponent 1, or multiplying by 1, or adding zero.²⁰ On the other hand, the assertion operator is without a doubt the only expression of symbolical notation which to a large extent renders the sense of the phrase “it is true that ...” It seems, then, that the acceptance of an operational reading of truth expressions entails the acceptance of ‘full blooded’ redundancy theory, for we can always dispense with an operator which does not change anything.

Here we have the answer to our question: Why, despite the syntactically suggested symmetry between the two expressions, namely “it is not true that ...” and “it is true that ...,” is the former easily read in the operational way and the latter is not? Our intuitive resistance to the idea that the proper logical sense of “it is true that ...” is expressed by the assertion operator has the same source as our resistance to redundancy theory. It springs from the conviction that the phrase “it is true that ...” (like the phrase “it is not true that ...”) may function as a semantically relevant component of our discourse. This conviction was brilliantly described by Frege in his interesting, though not a very conclusive, remark regarding truth:

It is ... worth noticing that the sentence “I smell the scent of violets” has just the same content as the sentence “it is true that I smell the scent of violets.” So it seems, then, that nothing is added to the thought by my ascribing to it the property of truth. And yet is it not a great result when the scientist after much hesitations and laborious researches can finally say “My conjecture is true”? The *Bedeutung* [meaning] of the word ‘true’ seems to be altogether *sui generis*.²¹

Frege recalled two contexts (from many possible ones) in which we use the word ‘truth.’ The use of this word in the first context is rather queer

¹⁹ A critique of the Fregean notion of assertion can be found in several places in the works of the early Wittgenstein, for instance in propositions 4.063 and 4.064 of the *Tractatus*.

²⁰ Notice that all of these mathematical operations are not meaningless in the sense that they are forbidden (like division by zero). They are simply useless; they bring nothing to our calculations.

²¹ Frege (1977: 328).

due to the special grammar of first-person expressions of feelings. We can replace the Fregean example by the following pair of sentences: “the book lies on the table” and “it is true that the book lies on the table.” We gain absolutely nothing by preceding the first sentence with the phrase “it is true that ...” though there is no grammatical prohibition – no rule embedded in our use of this expression – which would forbid us adding it to the first sentence. This observation seems to favor the redundancy theory.²²

However, the second context mentioned by Frege seems to indicate that the redundancy theory is wrong, since it ignores situations in which the sense of the phrase “it is true that ...” has an essential contribution to the sense of the whole. Note that the second Fregean example refers to a situation where a subject who formulates a conjuncture does not initially assign any definite logical value to it, for one does not yet know whether it is true or false. The knowledge has come after some time and it has been expressed by the words “My conjuncture is true.”

We believe that both contexts mentioned by Frege can be read in an operational manner. It is rather obvious in regard to the first. The trouble with the second context is the supposition that there is a third logical value, which can drive us back to the predicative theory of truth. Therefore, we must take a look at non-classical systems. The fact that a logical value of a sentence is undetermined can be rendered in a symbolic system by an ascription of an intermediate value to the sentence; Łukasiewicz was guided by this idea when introducing his three-valued logic. But when you admit three-valued logic, nothing can stop you from admitting infinitely many logical values.²³ In consequence, the logical value becomes a parameter and the truth (alternatively falsity) is understood just as a peculiar value of probability, namely 1 (alternatively 0); probability itself is understood as a concept more fundamental than truth, a measurable feature of sentences like weight or size of some objects. But such an interpretation trivialises both the concept of truth and the concept of probability. It seems that acceptance of many-valued systems entails intuitions which blur the uniqueness of the logical values.

²² Of course, provided that the sentence in question is part of non-anaphoric expression (if there are any), not part of a dialogue.

²³ Cf. Woleński (1989: 119–128).

So far, we have examined merely syntactical clues which can support our operational reading of truth expressions. But there are much more fundamental pragmatic reasons for accepting this view. Let us start with the question concerning the genesis of the concepts of truth and falsity. Considering this difficult subject matter, we appeal to an interesting conception of Alasdair MacIntyre. MacIntyre, in his *Whose Justice? Which Rationality?*, convincingly argues that the question of whether any belief is true is preceded by an experience of falsity.²⁴ Therefore, it can be said that the explicitly expressed concept of falsity is ‘older’ than the explicit concept of truth. Strictly speaking, the latter can be formed only on the model of the former – as its counterpart – to make an explicit expression of assertion possible, which was previously expressed only in an implicit way, namely, by uttering relevant indicative sentences.

The moment of emergence of the concept of falsity is extremely important for the analysis of sentences. Before this moment, sentences could have one of the following forms: Fx , xRy , or $P(x1, x2, x3\dots)$, and so on.²⁵ The expression of the experience of falsity consists in the extension of basic forms of indicative sentences – the extension enables us to say that a certain belief is false. Hence the forms have been enriched by the phrases equivalent to the English “it is false that ...” The logical structure of the new forms contains the operator of negation, or, to be more precise, a *place* for an operator which is initially filled by the operator of negation, but which can be filled later by any unary operator. The new forms can be represented as follows: ψFx , ψxRy , etc., where ψ is an operator variable. The domain of the operator variable is a set of unary sentential operators.²⁶

These considerations have led us to the point where we can explain more closely what is the source and the function of the assertion operator. Its possibility emerges with the possibility of negative sentences. The assertion operator can fill the place for negation and other unary sentential operators,

²⁴ Cf. MacIntyre (1988: 355–357).

²⁵ These are, respectively sentential functions from which one can obtain, by making relevant substitutions, the following sentences: “*Socrates* is a man,” “*Othello* loves *Desdemona*,” “*Alfred* needs a *car* in order to go from *London* to *Brighton*.” All arguments are italicised in these examples.

²⁶ The idea of an operator variable came to Łukasiewicz’s mind before the second world war. Cf. Woleński (1989: 107–110). According to us, this variable, contrary to Woleński’s belief, is of a great importance for the logical analysis of sentences.

such as “it is possible that ...” and “it is necessary that ...”, which enable us to express our affirmation of the content of indicative sentences. This way of affirmation is no longer ‘naïve’; it comes after the recognition of the possibility of denying any sentence. This possibility is inherent in the logical structure of a sentence extended by a place for an operator. Thus, from a pragmatic point of view, the vital use of the assertion operator does not consist in the transition from Fa to $\neg Fa$, as this transition is pragmatically idle, but rather consists in replacing other operators in sentences of the form: $\sim Fa$ or $\diamond Fa$. The most evident example of linguistic activity a couple of different unary sentential operators are used in its discussion – disputants can adopt different attitudes, expressed by the use of relevant operators, toward a certain view being considered.

We can see that although treating the expression “it is true that ...” as an operator seems to deprive it of an important semantic function, it plays a significant role, should it be considered in the context of molecular sentences. It finds its natural place in dialogical situations where different attitudes toward the same content are adopted. These various attitudes are expressed by unary operators like assertion, negation, possibility, etc.

MacIntyre’s thesis that the experience of falsity is prior to the experience of truth and its extension, saying that the concept of truth was formed later than the concept of falsity, can provoke objections. It seems that assertion is more fundamental than negation, at least for a reason that any negative sentence is more complex than corresponding positive sentences—the former consists of a indicative sentence and the specific operator while the latter may be expressed without using any operators. We do not want to reject it. Moreover, the claim that the idea of assertion is implicitly inherent in the idea of an indicative sentence is the climax of our considerations. However, according to our belief, the formation of the *explicit* concept of truth has become possible after the formation of the *explicit* concept of falsity. There is no contradiction: use of the indicative mood *implying* assertion of a content does not require *explicit* assertion of a content.

V

If we assume that ‘|-’ is an abbreviation of “it is true that ...,” then the following scheme expresses the right intuition underlying the Convention T:

6. $\vdash p \equiv p$

The meaning of the assertion operator, as we mentioned before, can be explained by means of the following truth table:

p	$\vdash p$
T	T
F	F

Look into this complex symbol for a moment. What, in fact, do its components stand for? Particularly, what is the meaning of the ‘T’ and ‘F’ signs? We admit that the standard notation of truth tables can suggest that they refer to some special logical objects. However, the truth tables should be preferably treated as a kind of heuristic device, just as they were by Wittgenstein when he used them in defining logical connectives for the first time.²⁷ We believe that the best way to understand the real content of truth tables is to translate them into a language which is closer to everyday speech. When we make such a translation, we get the explication of the meaning of a truth table for the assertion operator as follows: Any sentence p is true iff its assertion is true, and it is false iff its assertion is false. Now, this explication can be stated more systematically, namely:

7a. It is true that p iff it is true that $\vdash p$.

7b. It is not true that p iff it is not true that $\vdash p$.

These conditions can be transformed into the language of sentential calculus as follows:

8a. $\vdash p \equiv \vdash \vdash p$

8b. $\sim p \equiv \sim \vdash p$

Analogously the truth table for the ‘twin’ operator of negation can be explicated by the following formulas:

9a. $\vdash p \equiv \sim \sim p$

9b. $\sim p \equiv \vdash \sim p$

This translation is possible only if we reject the idea that the expression, “... is true” predicated of sentences in a given language must belong to

²⁷ The early Wittgenstein considered several different ways of defining logical constants. The task can be done, for example, in terms of ‘^ab p’ notation, presented in *Notes on Logic*, or by using the signs ‘T’ and ‘F.’ Cf. Wittgenstein (1979b: 102), Wittgenstein (1961: 4.443). The variety of possible notations supported his main idea, being as follows: “the ‘logical constants’ do not represent.” Wittgenstein (1961: 4.0312).

its meta-language. While translating, we must be wary, because it is easy to forget that the expression which is on the right side of the sign of the equivalence must have a more complex logical form than that which is on the left side – this feature should be reflected by the notation. Any notation which is not perspicuous enough to exhibit this difference can be a source of semantic paradoxes.²⁸

What has been achieved in this way? We have translated the truth table, in which there are symbols seeming to stand for logical objects (as we know, Frege was misled by that appearance), into expressions containing sentential variables preceded by operators. Some may say we have a problem here, since our procedure of dispensing with truth tables, which are treated as semantic definitions of operators, has led to pairs of sentences which are not normal definitions – they do not enable any substitution of assertion and negation by other expressions. They are axioms characterizing how symbols ‘|–’ and ‘~’ function. If there is any operator preceding a sentence, the result of the operation of negation applied to the whole is the sentence preceded by the operator opposite the original one, whereas the operation of assertion applied to the whole does not result in the change of the operator. This syntactic feature suggests – to those who roughly understand unary logical operators—that truth is the designated value.²⁹

What, however, does it mean that we understand these operators? It is a question of an interpretation of a symbolic system. As we pointed out, one can be deluded into thinking that one can interpret such a system only by means of its meta-language. In fact, as we argued, every meta-language is

²⁸ Our approach to this question is based on the following Wittgenstein’s claim: “No proposition can say anything about itself, because the propositional sign cannot be contained in itself (that is the whole ‘theory of types’)”. Wittgenstein (1961: 3.332). The first part of this statement suggests that it applies only to predicates, but a similar argument concerning expressions containing sentential operators can be formulated as well. This argument can be used to dissolve semantic paradoxes. (Wittgenstein shows, in the next proposition of *Tractatus*, that Russell’s paradox can be dispensed of in this way.) We will return to this point in the appendix to our article.

²⁹ This thought can be presented by means of the following logical riddle: Assuming that f and g are unary extensional operators, and:

- 1) $f \neq g$,
- 2) $f p \equiv f f p$,
- 3) $g p \equiv g f p$,

the question is: Which operator does f stand for? It is, of course, the assertion operator.

merely a translation of a relevant object language supplemented by rules associating primitive expressions of both languages—they are treated by adherents of formal semantics as rules of interpretation of an object language. A meta-language can give us what we want only if its interpretation is unproblematic, but there is no general reason to think that an understanding of an object language is more problematic than an understanding of its meta-language. Of course, some adherents of the idea of meta-languages do not admit it, but they do not present any reasonable arguments supporting their beliefs.

We can understand symbolic systems, such as sentential calculus, as an abstraction from natural language. The most important in that abstraction is a strict formulation of logical connections holding between sentences. But the very understanding of these connections, and hence the understanding of truth and falsity, relies on the understanding of a natural language possessed by competent users, no matter how inexplicit it is. Moreover, this understanding cannot be described from outside of any language. The concept of truth can be elucidated by examples which show how important true beliefs are for our life and how useful they can be. Imagine a hungry and determined buffalo hunter. If he believes that it is true that a big herd of buffalos is half a day's journey west from the place where he is, he will set off west and continue his journey at least for half a day, but if he believes that this information is false, he will not leave his home unless he has other reasons for going west (or in some other direction). Of course, the aim of our example is not to vindicate the so-called pragmatic definition of truth, for being true is obviously not the same as being useful. According to us, the concept of truth cannot be defined. We can only illustrate its use.

So, in what sense does the operational reading of truth expressions explain 'the nature' of truth? Only in the sense that it correctly explicates the syntactic structure of sentences having the following superficial forms: "it is true that *p*" and "the sentence '*p*' is true." It is obvious that it is not a definition of truth *simpliciter*, but only a partial elucidation of the concept.³⁰ As we already mentioned, the full explanation of the meaning of the assertion

³⁰ As Frege rightly emphasised, there are explanations which are not definitions, because what is logically simple and primitive cannot be defined, but only elucidated. This means that we can only give some hints or suggestions that will lead the reader to a proper grasp of a question. Cf. Frege (1984).

operator cannot be based on its syntactic features alone³¹—it must, of course, appeal to the use of this sign within different language-games. It is also worth pointing out that the claim that the concept of truth is indefinable *simpliciter* is not a hallmark of the operational conception of truth expressions. For example, Tarski observed that although it is possible to define the word ‘true’ for any given language (which satisfies certain conditions) in its meta-language, this word cannot be defined, should it express the concept of truth not relativised to any particular language.³²

Indefinability of truth *simpliciter* (alternatively, falsity *simpliciter*) refers to the fundamental characteristic of any sentence, namely, to its being a “propositional sign in its projective relation to the world.”³³ We cannot understand what the indicative mood is if we do not understand what truth and falsity are, and *vice versa*: We cannot understand what truth and falsity are if we do not understand what the indicative mood is.³⁴ The meaning of indicative sentences is determined by the conditions in which they are true and the ones in which they are false. Thus, the dichotomy of truth and falsity is an intrinsic feature of any sentence. They are spanned between their truth possibilities. This was excellently formulated by Wittgenstein:

³¹ Dummett, in his article *Truth*, points out, appealing to an analogy with chess, that you cannot fully describe the language-games of assertion in only syntactical terms. As there is a fundamental difference between chess and a game having the same rules, but differing in that the winning consists in being check-mated; so there is a vital dissimilarity between our language-games and some language-games with the same grammatical rules but the opposite goals, namely uttering false sentences. We should add, however, that in the case of language, the second option seems to be difficult to imagine or even illusory. See Dummett (1978).

³² Cf. Tarski (1983). The view that the concept of truth is not definable was accepted also by Frege and Davidson. Cf. Frege (1977), Davidson (1996).

³³ Wittgenstein (1961: 3.12).

³⁴ The late Wittgenstein expressed this in the following way: “And what a proposition is is in one sense determined by the rules of sentence formation (in English for example), and in another sense by the use of the sign in the language-game. And the use of the words ‘true’ and ‘false’ may be among the constituent parts of this game; and if so it *belongs* to our concept ‘proposition’ but does not *fit* it.” Wittgenstein (1974: §136). Cora Diamond, analyzing Wittgenstein’s remarks on truth, also notices that there is a dependence between the concept of truth and the concept of sentence. Cf. Diamond (2002), Diamond (2003).

From this it results that ‘true’ and ‘false’ are not accidental properties of a proposition, such that, when it has meaning, we can say it is also true or false: on the contrary, to have meaning means to be true or false: the being true or false actually constitutes the relation of the proposition to reality, which we mean by saying that it has meaning (*Sinn*).³⁵

Thus, truth and falsity are intrinsic and indefinable aspects of every meaningful indicative sentence³⁶—Wittgenstein tried to exhibit this fact by the use of different notations, e.g. truth tables.³⁷ These aspects are inseparable and cannot be detached from any sentence—it is not possible to ask about their nature without asking about the essence of sentence. However, they are not symmetrical, because every sentence has the claim to be true.³⁸ We formulated the content of this claim earlier in the following way: the idea of assertion is implicitly inherent in the idea of indicative sentences. The implicitly inherent idea of assertion precedes every operation of cancelling it. A sentence is not so much a line spanned between the poles of truth and falsity—to use a metaphor taken from Wittgenstein’s *Notes on Logic*—but an arrow which points out one of these poles, namely truth.³⁹

The equivalence of both columns of the truth table for the assertion operator shows that the operator does not bring into a language anything more than what is already contained in an indicative sentence. This statement can be interpreted twofold. It can mean that the concept of truth is superfluous. In our opinion such a thesis is unjustified. We believe that the assertion operator can be useful in explicating the claim to be true implied by any indicative sentence. It does not mean that there is such a property as being true (unless we construct a certain property and call it ‘truth’)—in our analyses of natural language sentences containing phrases like “it is true that ...” and “... is

³⁵ Wittgenstein (1979c: 113). We do not change the traditional translation of the German word *Satz* in this and the following quotations from Wittgenstein.

³⁶ We are aware that saying “truth and falsity are aspects of sentence” sounds like saying that they are properties, perhaps very special, but properties. So, is our treating the expression “it is true that ...” as an operator—to use Diamond’s phrase—chickening out? We use this transitional way of speaking because we think that it can help to avoid deflationism, which is in a sense the other side of the correspondence theory of truth.

³⁷ See footnote 27.

³⁸ Cf. Wittgenstein (1979c: 113).

³⁹ “Names are points, propositions arrows—they have sense. The sense of a proposition is determined by the two poles, true and false.” Wittgenstein (1979b: 101–102).

true,” we try to show that the logical form and function of such expressions radically differs from that of predicates. Contrary to deflationists, however, we claim that the concept of truth plays a not only special, but also significant role in our language, and hence in our lives. Briefly speaking, the difference between deflationism and our conception consists in the fact that deflationists hold that the concept of truth can be in principle eliminated from all contexts, while we claim that this concept is indispensable in the language-games where we are free to question the natural, implicit claim of the sentences we use to be true.

Note that the concept of truth—despite a view which is widespread among philosophers and which is mentioned in section III—is not originally applied to sentences. This means it is not originally *predicated* of them, but *connected* with them. That is why it is used to speak about a reality described by a sentence, not about the sentence itself. So, there is a closer likeness between the concept of truth and the concept of being than we suggested earlier. Look at the following utterances: “*something* exists” and “it is true that *thus and so*.” The expression “thus and so” functions in this context analogously to the pronoun ‘something’: The former plays the role of an individual variable, the latter of a sentential variable. We replace the relevant variables with names and sentences, but the semantic values of names and sentences are, of course, extra-linguistic entities. When we say that there are dragons, we do not mean that there is the name ‘dragon,’ but that there are certain creatures. Similarly, when we say “It is true that the sun is shining today,” we do not mean that the subordinate sentence has a certain property, namely, a property of being true. What we mean is that some state of affairs obtains, namely, that the sun is shining today.

In brief, the words ‘truth’ and ‘true’ are used in order to point out that what we are speaking about is ultimately an extra-linguistic reality.⁴⁰ Moreover, it is possible to substitute in certain contexts the word ‘truth’ by the word ‘reality.’ We can say that the reality is such that there is a time for everything instead of saying that the truth is such that there is a time for everything. This does not mean that we should accept the medieval view that the concepts of truth and being are interchangeable,⁴¹ for such

⁴⁰ This view is accepted by, for example, Willard Van Orman Quine. See: Quine (1970).

⁴¹ A similar approach has recently been developed by adherents of the identity theory of truth; cf. Hornsby (1997).

a substitution is impossible in many contexts. The statement “the reality which surrounds us is full of useless things” is not only meaningful, but also true; on the other hand, it is difficult to assign any sense to the utterance: “the truth which surrounds us is full of useless things.” The significance of the concept of truth does not consist in its representing a kind of entity: an object, property or relation. A deeply rooted inclination tempts us to think that by rejecting the naming function of the word ‘truth,’ we deprive the word of any significance.⁴² In fact, we reject only an illusion concerning its function in language. Truth is neither something located in the world nor something to be found in mind or language. This illusory alternative leads to vehement disputes concerning the question of which kind of entity truth is. However, we think that the best way to determine the essence of truth is to look at the grammar (in the later Wittgenstein’s sense) of the expressions “it is true that ...” and “... is true”.

The operational interpretation of truth expressions shows what connection between truth and reality consists in. First, the fact that truth expressions are concatenated with sentences and not with their names explains why the subject of a statement assigning truth to a sentence is not the sentence itself, but what it speaks about. The use of the operator does not serve to assert something about the sentence which is the basis of the operation, but, at most, it can modify the content of the sentence. Second, according to our conception, there is a relation of mutual entailment between sentences of the form $\neg p$ and p . It is warranted by the truth table for the assertion operator. When we say it is true that there is a time for everything, we thereby assert something about reality, namely that there is a time for everything. (It sounds trivial, but it is doubtful whether it is possible to say something really original as far as this question is concerned.) In other words, if the fact that any sentence says that things are thus and so belongs to its essence, and if any sentence of the form $\neg p$ entails a sentence of the form p , then the sentence of the form $\neg p$ also says that things are thus and so. Therefore, our conception is not liable to the objection which we raised against a predicative reading of truth expressions, because it explicates

⁴² The other reason for rejecting the operational interpretation of truth expressions may be the justified belief that it precludes formulation of a normal definition of truth for any language in which it is possible to construct infinitely many sentences. It seems, however, that this reason is insufficient, because the thesis that the concept of truth can be defined is not supported by undeniable arguments—it is rather a demand.

in a perspicuous way an intuitively accepted necessary character of the connection between the left and right side of equivalences such as our sentence 1.

Conclusion

The approach to the notion of truth presented above is not an alternative to the classical conception, if the latter is understood in the following way: “to say that what is is, and what is not is not, is true.”⁴³ The main advantage of the operational reading is, in our opinion, that it represents in an adequate way how the expressions “it is true that ...” and “... is true” work in everyday speech. Our conception confirms a widespread intuition that so-called T-equivalences are not contingent truths, and shows why it is so. This enables us to avoid difficulties related to the belief that there is a peculiar and mysterious property, namely, being true. While rejecting the belief that truth is a property, we do not deny the significance of the concepts of truth and falsity. We indicate that the notions of truth and sentence are in a sense inseparable. You cannot explain what sentence is without an appeal to the concept of truth, nor can you explain what truth is without any reference to the concept of sentence or the notion of its content. It is worth noticing that we believe that the concept of falsity is, from a pragmatic point of view, even more significant than the concept of truth. Statements of the form “it is true that p ” are not, from a pragmatic point of view, derivatives of utterances of the form “ p ,” but strictly connected to the sentences of the form “it is false that p .”

Appendix

It is a widespread opinion that the most important criterion for a test indicating whether a conception of truth is correct or not is its ability to eliminate the liar paradox. Tarski’s approach, which we have criticised, passes the test. Does our conception pass it, too? If not, then, even if it were in all other respects better than the predicative reading, it should be

⁴³ Aristotle, *Metaphysics*, VII 7, 1011 b. Cf. Aristotle (1933: 201).

rejected. It can seem, at first sight, that our conception cannot deal with this antinomy. We believe, however, that irrespectively of which account of a logical form of truth expressions we adopt, the so-called liar sentence generates only an apparent problem. Moreover, the operational reading has an advantage because it enables one to notice almost immediately how difficult it is to assign any determinate meaning to the liar sentence. The real form of statements such as “the sentence ‘snow is white’ is true” is not the sentential function ‘ $P(x)$ ’, in which ‘ x ’ should be substituted by a name of the sentence and ‘ $P(\dots)$ ’ by the predicate “... is true,” but the sentential function ψp , in which ‘ p ’ should be substituted by the sentence and ‘ ψ ’ by the operator “it is true that ...” A sentence containing the operator “it is true that ...” or “it is false that ...” must contain its base as well. And the base can contain the operator, too, but there must be a primitive, non-molecular sentence at last. If there were not, the sentence in question would be infinitely complex.

The parallel conclusion from the predicative standpoint is not obvious at first blush. If the names of sentences, and not the sentences themselves, appear in statements assigning truth or falsity to those sentences, then the belief that the ultimate objects of our statements must be sentences which do not contain semantic predicates requires an additional justification.⁴⁴ Our conception easily explains the intuition that the ultimate objects of our statements must be sentences which do not contain semantic terms. Apply the above observations to the following string of words:

11. “11 is false.”

How should we read this utterance? If we appeal to the operational reading of truth expressions, we will get the infinite series:

12. “It is false that it is false that it is false that ...”

⁴⁴ This belief, however, is rarely supported by an independent argument. It is often justified by observing that its rejection leads to the liar paradox. Adherents of a predicative reading put forward different proposals for avoiding this antinomy. Tarski’s classical solution assumes that utterances of everyday language—if they have any definite meaning at all – do not belong to one language, but to a hierarchy of languages of which the lowest element is a language lacking semantic terms. Another proposal was advanced by Saul Kripke. See: Kripke (1975). Kripke’s conception is based on the postulate of groundness – the postulate which is in a sense a counterpart of the belief that the ultimate objects of our statements must be sentences which do not contain semantic predicates. Kripke appeals to this requirement because he believes that it is in accordance with our intuitions.

Our conception requires the substitution of the symbol ‘11’ by the sentence 11. (According to the standard approach it should be substituted by the sentence 11 enclosed by quotation-marks, but it does not matter whether we accept the standard conception at this point, because the expression, “the sentence ‘...’ is false”—from the point of view of an operational reading – plays the same role as the expression “it is false that ...” —namely, the role of an operator. What we do need to reject is the claim that a sentence put into quotation marks must be its own name. The fixation of the logical form of the liar sentence cannot be finished, because each occurrence of the symbol ‘11’ must be substituted by the sentence “11 is false,” which contains this very symbol. No sentence, however, can have an infinitely complex logical form, so – if we try to interpret it in the most natural way – we cannot assign any logical form to it. Taking into account that no definite meaning is given to sentences no logical form is assigned to, it has to be admitted that, despite appearances, no definite sense is given to the liar sentence.

It is worth noticing that the liar sentence raises a merely apparent difficulty for adherents of predicative conceptions as well. The sentence seems to be obtained by a substitution of the formula $X(p)$, in which ‘ $X()$ ’ is a variable with a domain of unary predicates and ‘ p ’ is a sentential variable. ‘ $X()$ ’ is substituted by the predicate “... is false,” but what should be inserted in the place of ‘ p ’? The liar sentence, of course. This step, however, does not stop the process of the analysis of the liar sentence’s logical form, because, as we assumed, the sentence just inserted seems to have the form $X(p)$, so it turns out that the liar sentence has the form $X(X(p))$. Thus, relevant expressions must be put again in place of the variables—the result of this substitution is a sentence of the form $X(X(X(p)))$. The procedure can never end; it drives us, of course, to an infinite regress.

References

- Anscombe, G. E. M. 2000. “Making True.” In: R. Teichmann (ed.). *Logic, Cause & Action. Essays in Honour of Elizabeth Anscombe*. (Royal Institute of Philosophy, *Philosophy* Supplement: 46). Cambridge: Cambridge University Press, pp. 1–8.
- Aristotle 1933. *Metaphysics*. Trans. H. Tredennick. London & Cambridge, MA: Heinemann & Harvard University Press.

- Brandom, R. B. 1994. *Making It Explicit: Reasoning, Representing, and Discursive Commitment*. Cambridge, MA: Harvard University Press.
- Davidson, D. 1996. "The Folly to Define Truth." *Journal of Philosophy* 93: 263–278.
- Diamond, C. 2002. "Truth before Tarski: After Sluga, after Ricketts, after Geach, after Goldfarb, Hylton, Floyd, and Van Heijenoort". In: E. Reck (ed.). *From Frege to Wittgenstein: Perspectives on Early Analytic Philosophy*. Oxford & New York: Oxford University Press, pp. 252–279.
- Diamond, C. 2003. "Unfolding Truth and reading Wittgenstein". *Nordic Journal of Philosophy* 4: 24–58.
- Dummett, M. 1973. *Frege—Philosophy of Language*. London: Duckworth.
- _____. 1978. "Truth." In: M. Dummett, *Truth and Other Enigmas*. Cambridge, MA: Harvard University Press, pp. 1–25.
- _____. 2000. "Sentences and Propositions." In: R. Teichmann (ed.). *Logic, Cause & Action. Essays in Honour of Elizabeth Anscombe* (Royal Institute of Philosophy, *Philosophy Supplement*: 46). Cambridge: Cambridge University Press, pp. 9–23.
- Frege, G. 1970. *Begriffsschrift, a Formula Language, Modeled upon that of Arithmetic, for Pure Thought*. Trans. S. Bauer-Mengelberg. In: J. van Heijenoort (ed.). *Frege and Gödel: Two Fundamental Texts in Mathematical Logic*. Cambridge, MA: Harvard University Press, pp. 1–82.
- _____. 1977. "Thought." Trans. M. Beaney. In: M. Beaney (ed.). *The Frege Reader*. Oxford: Blackwell, pp. 325–345.
- _____. 1984. "On Concept and Object." Trans. P. Geach. In: G. Frege, *Collected Papers on Mathematics, Logic, and Philosophy*. Ed. B. McGuinness. Oxford: Blackwell 1984, pp. 182–194.
- Geach, P. 1968. *Reference and Generality. An Examination of Some Medieval and Modern Theories*. Ithaca & London: Cornell University Press.
- Geach, P. 1971. *Mental Acts: Their Content and Their Objects*. London & New York: Routledge and Kegan Paul.
- Grover, D., Camp, J., Belnap, N. 1975. "A Prosentential Theory of Truth." *Philosophical Studies* 27: 73–125.
- Heijenoort, J. van 1967. "Logic as Calculus and Logic as Language." In: *Boston Studies in the Philosophy of Science*. Eds. R. S. Cohen and M. W. Wartofsky. Dordrecht: Reidel, pp. 440–446.

- Hornsby, J. 1997. "Truth: The Identity Theory". *Proceedings of the Aristotelian Society* 97: 1–24.
- Kant, I. 1998. *Critique of Pure Reason*. Trans. P. Guyer and A. W. Wood. Cambridge: Cambridge University Press.
- Kripke, S. 1975. "Outline of a Theory of Truth." *Journal of Philosophy* 72: 690–716.
- MacIntyre, A. 1988. *Whose Justice? Which Rationality?* London: Duckworth.
- Quine, W. V. O. 1970. *Philosophy of Logic*. Englewood Cliffs, NJ: Prentice-Hall.
- Tarski, A. 1983. "The Concept of Truth in Formalized Languages." In: A. Tarski, *Logic, Semantics, Metamathematics, Papers from 1923 to 1938*. J. Corcoran (ed.). Indianapolis: Hackett, pp. 152–278.
- Wittgenstein, L. 1961. *Tractatus Logico-Philosophicus*. Trans. D. Pears and B. McGuinness. London: Routledge and Kegan Paul.
- _____. 1974. *Philosophical Investigations*. Trans. G. E. M. Anscombe. Oxford: Blackwell.
- _____. 1979a. *Ludwig Wittgenstein and The Vienna Circle. Conversations recorded by Friedrich Waismann*. Trans. J. Schulte and B. McGuinness. Oxford: Blackwell.
- _____. 1979b. *Notes on Logic*. Trans. G. E.M. Anscombe. In: L. Wittgenstein, *Notebooks 1914–1916*, 2nd edition. G. H. von Wright and G. E. M. Anscombe (eds.). Oxford: Blackwell, pp. 93–107.
- _____. 1979c. *Notes dictated to G.E. Moore in Norway, April 1914*. Trans. G. E. M. Anscombe. In: L. Wittgenstein, *Notebooks 1914–1916*, 2nd edition. Ed. G. H. von Wright and G. E. M. Anscombe. Oxford: Blackwell, pp. 108–119.
- Woleński, J. 1989. *Logic and Philosophy in The Lvov-Warsaw School*. Dordrecht & Boston & London: Kluwer.